K-k-ε Zone Model of In-Cylinder Turbulence Generated from Bulk Flow

Computer time consuming full 3-D CFD models should be used with understanding of main issues of turbulence modelling. That is why the simpler Q-D (zone) models are still worthwhile either for the first approximation during early stages of engine development or as an extrapolation of basic CFD simulations during optimization phase. They are based on cascade of kinetic energy changes from bulk flow to smaller and chaotic turbulence motion and finally its decay to internal energy.

Time-rate of in-cylinder kinetic energy is composed of generation $\boldsymbol{G_{in,i}}$,, extracting power from a piston, transformation – sink term – of bulk flow kinetic energy into turbulence kinetic energy $\boldsymbol{g_{in,i}}$ and convection of kinetic energy (especially convection into a cylinder by injection of fuel or out of a cylinder due to exhaust flow, e.g., during scavenging). Turbulent kinetic energy $\boldsymbol{k_{in,i}}$ is created by dampening bulk flow kinetic energy.

$$\frac{dK_{in,i}}{dt} = G_{in,i} - g_{in,i} + \dot{m}_{ext,i}\frac{\overline{w_{ext,i}^{\;2}}}{2} - \dot{m}_{out}\frac{K_{in}}{m_{in}}$$

$$\frac{dk_{in}}{dt} = \sum g_{in,i} + \dot{m}_{ext,i}\frac{\overline{3w_{ext,i}'^{\;2}}}{2} - \dot{m}_{out}\frac{\overline{3w'^{\;2}}}{2} - m_{cyl}\,\varepsilon$$

Time-rate of kInetic energy of bulk flow $\boldsymbol{K}_{in,i}$ can be found by adding angular momentum of appropriate tangential velocity components, rotating along appropriate vortex axis. Rotation movements inside a cylinder are assumed as rotations of a solid body (cores of vortices fill the almost the whole vortex ranges). Toroidal squish and swirl rotating along cylinder axis feature tangential velocity components in perpendicular planes. It means, kinetic energies can be simply added. Tumble spherical vortex is not combined with toroidal squish (anti-knock, piston-crown-head clearances are oriented to support squish) but may be combined with swirl. Again, the vortex tangential components of velocities are perpendicular each to other in this case. Then, time-rate of a component of kinetic energy belonging to a single vortex is, respecting the variability of both angular speed and moment of inertia due to changes of cylinder dimensions and density inside it, yields

$$\frac{dK_{in,i}}{dt} = \frac{1}{2}\frac{dI_i\omega_i^2}{dt} = I_i\omega_i\frac{d\omega_i}{dt} + \frac{\omega_i^2}{2}\frac{dI_i}{dt}$$

Bulk motion kinetic energy generators $\boldsymbol{G_{in,i}}$ depend on additional piston power, caused by delivering all components of kinetic energy to flow (e.g., through inlet valve – even without any organized swirl – or through a clearance between a piston periphery and cylinder head in squish area). In the case of swirl dimension change, kinetic energy rises due to angular momentum conservation. This change is covered by increased piston work due to additional pressure caused by centrifugal force. The generation of turbulent kinetic energy $\boldsymbol{g_{in,i}}$ can be found from equation () if the additional work of a piston is calculated from averaged in-cylinder flows induced by piston motion and if angular speed of in-cylinder vortices in equation () is calculated by integration of angular momentum equation of motion – see below, ()

$$G_{in,i} = P_{PIST,i} = \dot{m}_{in,i} \frac{\overline{w_{in,i}^2}}{2} + \Delta p_C A_P \frac{dx}{dt}$$

$$g_{in,i} = P_{PIST,i} + \dot{m}_{ext,i} \frac{\overline{w_{ext,i}^{\;2}}}{2} - \frac{dK_{in,i}}{dt} - \dot{m}_{out} \frac{K_{in}}{m_{in}}$$

Equation for angular momentum component along appropriate axis with convection of angular momentum (tangential velocities averaged for reference radius) and with torque braking rotation by gas internal friction yields

$$\frac{dB_i}{dt} = \frac{d(I_i \omega_i)}{dt} = \dot{m}_{ext,i} \overline{w_{ext,t,i} r_{ext,i}} - \dot{m}_{out,i} \overline{w_{out,t,i} r_{out,i}} - M_{loss}$$

Friction loss torque in wall boundary layers of transversal length $\boldsymbol{L_i}$ at radius $\boldsymbol{R_i}$ with loss coefficient ζ_i , which will be calibrated by comparison to angular momentum sources from in-cylinder movements and depends on in-cylinder Re, is

$$M_{loss,i} = \frac{\Delta h_{loss,i} \rho_{cyl} A_i \omega_i R_i}{\omega_i} = \zeta_i \frac{\omega_i^2 R_i^2}{2} \rho_{cyl} R_i L_i \frac{R_i}{2} = \zeta_i \frac{\omega_i^2 R_i^4}{4} L_i \rho_{cyl}$$

In the case of a piston crown or a cylinder head, length is wetted perimeter. Angular momentum differential equation yields

$$\omega_i \frac{dI_i}{dt} + I_i \frac{d\omega_i}{dt} = \dot{m}_{ext,i} \overline{w_{ext,t,i} r_{ext,i}} - \dot{m}_{out,i} \overline{w_{out,t,i} r_{out,i}} - \zeta_i \frac{\omega_i^2 R_i^4}{4} L_i \rho_{cyl}$$

For time-rate of angular speed in energy equation () we have

$$\omega_i I_i \frac{d\omega_i}{dt} = \omega_i \dot{m}_{ext,i} \overline{w_{ext,t,i} r_{ext,i}} - \omega_i \dot{m}_{out,i} \overline{w_{out,t,i} r_{out,i}} - \zeta_i \frac{\omega_i^3 R_i^4}{4} L_i \rho_{cyl} - \omega_i^{\;2} \frac{dI_i}{dt}$$

which means for internal bulk flow kinetic energy time-rate

$$\frac{dK_{in,i}}{dt} = \omega_i \dot{m}_{ext,i} \overline{w_{ext,t,i} r_{ext,i}} - \omega_i \dot{m}_{out,i} \overline{w_{out,t,i} r_{out,i}} - \zeta_i \frac{\omega_i^3 R_i^4}{4} L_i \rho_{cyl} - \frac{\omega_i^2}{2} \frac{dI_i}{dt}$$

Finally, for turbulence kinetic energy source term combining equations for convection of kinetic energy from and to external domains and bulk flow inside. For higher Re the production of turbulence is increased due to vortex loss term.

$$g_{in,i} = \dot{m}_{in,i}\frac{\overline{w_{in,i}^2}}{2} + \dot{m}_{ext,i}\frac{\overline{w_{ext,i}}^2}{2} - \omega_i \dot{m}_{ext,i}\,\overline{w_{ext,t,i} r_{ext,i}} + \omega_i \dot{m}_{out,i}\,\overline{w_{out,t,i} r_{out,i}} +$$

$$+\zeta_i \frac{\omega_i^3 R_i^4}{4} L_i \rho_{cyl} + \frac{\omega_i^2}{2}\frac{dI_i}{dt} - \dot{m}_{out}\frac{K_{in}}{m_{in}}$$

Sink term for turbulence kinetic energy can be found from estimates with turbulent integral scale – []. Integral length scale is estimated from cylinder minimum dimensions, then turbulent energy dissipation rate is

$$\varepsilon = \frac{c_z}{l}\left(\frac{3\,\overline{w'^2}}{2}\right)^{\frac{3}{2}}$$

$$l = c_L \min\left(\frac{D}{2};\frac{4V_{cyl}}{\pi D^2}\right)$$

Calibration constants are usually found by experiments and follow basic features of old k-ε turbulence model; ***c_z*** between 0.05 and 0.28, ***c_L*** between 0.1 and 0.4. Surprisingly, the sink term does not depend directly on Re, since it is based on inertial part of turbulence spectrum dissipation. The smallest turbulent eddies with Re close to 1 (Kolmogorov scale) automatically change their dimensions in such a way that the inertial part of spectrum is finished by them. It means that for higher Re of bulk flow the Kolmogorov eddies are tinier, being out of scales simulated even by direct numerical simulation methods (DNS).

The result of turbulence modelling is averaged fluctuating turbulent velocity, suitable, e.g., for prediction of flame propagation speed or heat transfer correlations

$$k_{in,i} = 3\frac{\overline{w'^2}}{2} \Rightarrow \overline{w'} = \sqrt{\frac{2}{3}k_{in,i}}$$

The above mentioned general equations can be applied for different vortex motions inside a cylinder, as described below.

Squish

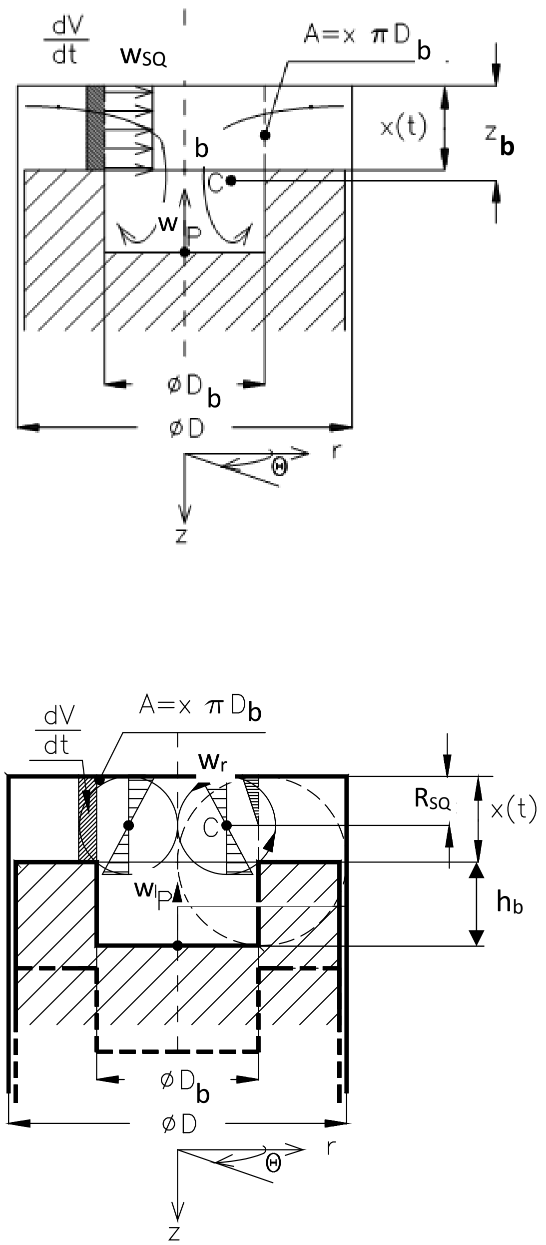

Figure 1 Squish motion and approximation of squish vortex radius

Radius of squish vortex reaching cylinder axis changes during piston stroke, starting at half bore and reaching minimum of bowl radius or half of its depth, which is less (

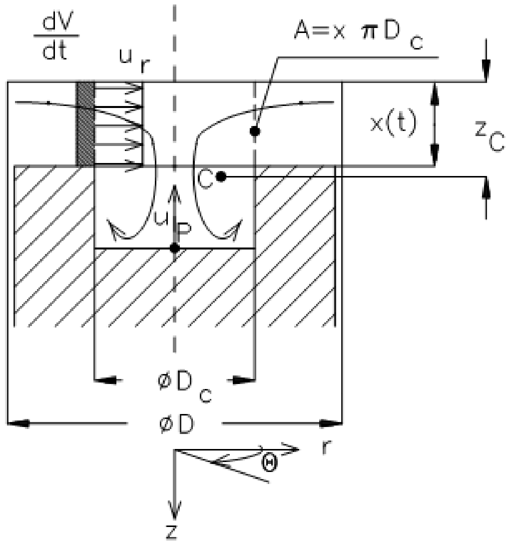

Figure 1)

$$r_b = \frac{D_b}{2}; \quad R = \frac{D}{2}; \quad A_P = \frac{\pi D^2}{4}; \quad A_b = \frac{\pi D_b^2}{4}$$

$$R_{SQ\max}^2 = (x - R_{SQ\max})^2 + (r_b - R_{SQ\max})^2$$

$$R_{SQ\max} = \frac{D_b}{2} + x - \sqrt{D_b x}$$

$$R_{SQ} = \left\langle \begin{array}{l} \text{if } \mathrm{R}_{\mathrm{SQmax}} < x \text{ then } \min\left(\frac{D_b}{4}; \frac{h_b + x}{2}; R_{SQ\max}\right) \\ \qquad \text{else } \min\left(R_{SQ\max}; \frac{D}{4}; \frac{h_b + x}{2}\right) \end{array} \right.$$

Squish motion with uniform velocity profile along $\boldsymbol{R_{SQ}}$ by redistribution of mass between squish area and bowl of $\boldsymbol{V_b}$ volume assuming the same pressure and temperature, i.e., same density in cylinder, yields

$$\frac{m_b}{m_{cyl}} = \frac{(V_b + A_b x)}{V_{cyl}}$$

$$\frac{dm_b}{dt} = \dot{m}_{SQ} = \frac{m_{cyl}}{V_{cyl}{}^2}\left(V_{cyl} A_b \frac{dx}{dt} - (V_b + A_b x)\frac{dV_{cyl}}{dt}\right) = \rho_{cyl} A_b \left(1 - \frac{h_b + x}{x + h_b \frac{A_b}{A_D}}\right) w_P$$

Uniform velocity of squish motion is approximately using flow through either half of vertical dimension of squish or distance of piston-head, which is less

$$A_{SQ} = 2\pi \frac{D_b}{2} \min\left(R_{SQ}; x\right)$$

As all the other geometric relations in this part, it is an approximation of the cross section of a flow generating tumble only. Especially at the large distance of a piston from cylinder head the real in-cylinder speed may be different but in thís case it has no real effect since the generating kinetic energy is very low. Then, the mean velocity generating squish is

$$w_{SQ} = \frac{\dot{m}_{SQ}}{\rho_{cyl} A_{SQ}} = \frac{A_b\left(1 - \frac{h_b + x}{x}\right)}{A_{SQ}} \frac{dx}{dt}$$

Volume and moment of inertia for toroidal shape relative to circular axis C is

$$V_{SQ} = 2\pi^2 r_{SQ}^2 R_{SQ} = 2\pi^2 R_{SQ}^3$$

$$I_{SQ} = \frac{1}{2} m R_{SQ}^2 = \rho_{cyl} \pi^2 R_{SQ}^5$$

The mass and moment of inertia time rate is (approximately – toroidal shape cannot rotate as solid body without redistribution of mass during rotation)

$$\frac{dm_{SQ}}{dt} = 2\pi^2 R_{SQ}^3 \frac{d\rho_{cyl}}{dt} + 6\pi^2 \rho_{cyl} R_{SQ}^2 \frac{dR_{sQ}}{dt} = -2\pi^2 R_{SQ}^3 \frac{m_{cyl} A_P}{V_{cyl}^2} \frac{dx}{dt} + 6\pi^2 \rho_{cyl} R_{SQ}^2 \frac{dR_{sQ}}{dt}$$

$$\frac{dI_{SQ}}{dt} = \pi^2 R_{SQ}^5 \frac{d\rho_{cyl}}{dt} + 5\pi^2 \rho_{cyl} R_{SQ}^4 \frac{dR_{sQ}}{dt} = -\pi^2 R_{SQ}^5 \frac{m_{cyl} A_P}{V_{cyl}^2} \frac{dx}{dt} + 5\pi^2 \rho_{cyl} R_{SQ}^4 \frac{dR_{sQ}}{dt}$$

Time rate of RSQ is zero if vortex radius is fixed by cylinder or bowl diameter, piston velocity if the vortex is limited by bowl depth and piston distance to a cylinder head or by derivative of ()

$$\frac{dR_{SQ\max}}{dt} = w_P \left(1 - \frac{\sqrt{D_b}}{2\sqrt{x}}\right)$$

Lost torque

$$M_{loss,i} = \zeta_i \frac{\omega_{SQ}^2 R_{SQ}^2}{2} \rho_{cyl} 2\pi R_{SQ} R_{SQ} \frac{R_{iSQ}}{2} = \zeta_i \frac{\pi \omega_{SQ}^2 R_{SQ}^5}{2} \rho_{cyl}$$

$$O = 2\pi R_{SQ} \; ; \quad A = 2\pi R_{SQ}^2$$

$$L = 2R_{SQ} \; ; \quad d_h = \frac{4A}{O} = 4R_{SQ}$$

$$\zeta = \frac{K}{\mathrm{Re}^n} \frac{L}{d_h} = \frac{K}{2\,\mathrm{Re}^n}$$

Angular momentum input of squish motion with initially uniform velocity profile along ***R**$_{SQ}$* yields

$$\frac{dB_{SQ}}{dt}=\dot{m}_{SQ}w_{SQ}\frac{\int_0^{R_{SQ}}rdr}{R_{SQ}}-\zeta_i\frac{\omega_i^3R_i^4}{4}L_i\rho=\dot{m}_{SQ}w_{SQ}\frac{R_{SQ}}{2}-\zeta_{SQ}\frac{\omega_{SQ}^2R_{SQ}^4}{4}2\pi R_{SQ}\rho_{cyl}$$, i.e.,

$$\omega_{SQ}I_{SQ}\frac{d\omega_{SQ}}{dt}=\omega_{SQ}\dot{m}_{SQ}w_{SQ}\frac{R_{SQ}}{2}-\frac{\pi\zeta_{SQ}}{2}\rho_{cyl}\omega_{SQ}^3R_{SQ}^5+\omega_{SQ}^2\left(\pi^2R_{SQ}^5\frac{m_{cyl}A_P}{V_{cyl}^2}\frac{dx}{dt}-5\pi^2\rho_{cyl}R_{SQ}^4\frac{dR_{sQ}}{dt}\right)$$

Then, squish kinetic energy rate is

$$\frac{dK_{in,SQ}}{dt}=\omega_{SQ}\dot{m}_{SQ}w_{SQ}\frac{R_{SQ}}{2}-\frac{\pi\zeta_{SQ}}{2}\rho_{cyl}\omega_{SQ}^3R_{SQ}^5+\frac{\omega_{SQ}^2}{2}\left(\pi^2R_{SQ}^5\frac{m_{cyl}A_P}{V_{cyl}^2}\frac{dx}{dt}-5\pi^2\rho_{cyl}R_{SQ}^4\frac{dR_{sQ}}{dt}\right)$$

and piston work overcomes inertia and centrifugal forces; bulk flow energy sink term is

$$\frac{dK_P}{dt}=\frac{dW_{PIST,SQ}}{dt}+\left|\dot{m}_{SQ}\right|\frac{w_{SQ}^2}{2}=\dot{m}_{SQ}\omega_{SQ}^2\frac{R_{SQ}^2}{2}+\left|\dot{m}_{SQ}\right|\frac{w_{SQ}^2}{2}$$

$$g_{SQ}=\frac{dK_P}{dt}-\omega_{SQ}\dot{m}_{SQ}w_{SQ}\frac{R_{SQ}}{2}+\frac{\pi\zeta_{SQ}}{2}\rho_{cyl}\omega_{SQ}^3R_{SQ}^5-\frac{\omega_{SQ}^2}{2}\left(\pi^2R_{SQ}^5\frac{m_{cyl}A_P}{V_{cyl}^2}\frac{dx}{dt}-5\pi^2\rho_{cyl}R_{SQ}^4\frac{dR_{sQ}}{dt}\right)$$

Swirl

Angular speed of swirl can be integrated during inlet stroke adding contributions of swirl flow through inlet valve. During a steady flow test, swirl contribution is characterized by a swirl number ***SWN***, calculated from equivalent engine speed at steady flow, i.e., from equivalent mean piston speed:

$$dI_{test}=\frac{\pi\rho_{cyl}}{2}\frac{D^2}{4}\frac{\dot{m}_{IO}}{\frac{\pi D^2}{4}\rho_{cyl}}dt\frac{D^2}{4}=\frac{\pi}{32}\rho_{cyl}D^4\frac{Zn_M}{30}dt$$

$$\frac{dB_{test}}{dt}=\omega_{test}\frac{dI_{test}}{dt}=SWN\frac{\pi n_M}{30}\frac{\pi}{32}\rho_{cyl}ZD^4\frac{n_M}{30}=SWN\frac{\pi^2}{32}\rho_{cyl}ZD^4\frac{n_M^2}{900}$$

During inlet stroke, the time-rate of inlet flow contribution to angular momentum in a cylinder is at averaged diameter of a swirl vortex

$$\frac{dB_{IO}}{dt}=\dot{m}_{IO}\overline{w_{IO,t,SW}r_{ext,SW}}=SWN\frac{\pi n_M}{30}\frac{D^2}{8}\dot{m}_{IO}$$

whereas ***SWN*** and mass flow rate are functions of valve stroke and piston position.

Swirl or tumble creates pressure field due to centrifugal force in a cylinder, which influences piston power by $\frac{dx}{dt}\sum A_P\Delta p_C$ (piston consumes it during inlet, delivers it during compression, etc.)

$$dF_C = \omega_{SW}^2 r\,\rho_{cyl}\,zrd\varphi dr$$

$$pzrd\varphi + pd\varphi\,zdr - (p+dp)z(r+dr)d\varphi + dF_C = 0$$

$$-rdp + \omega_{SW}^2 r\,\rho_{cyl} rdr = 0 \Rightarrow \frac{dp}{dr} = \omega_{SW}^2 r\,\rho_{cyl}$$

$$\Delta p_C = \omega_{SW}^2 \rho_{cyl}\,\frac{r^2}{2}$$

$$\Delta F_C = \int_0^{D/2} \Delta p\;2\pi r dr = \pi\,\frac{\omega_{SW}^2 \rho_{cyl}}{4}\,\frac{D^4}{16}$$

Then, using friction torque force at half of cylinder radius and swirl pipe as cross-section of half bowl and area between cylinder head and piston crown with longitudinal length of bowl perimeter

$$M_{SW,loss} = \frac{K}{\mathrm{Re}^n}\,\frac{L_{long}O}{4A}\,\rho_{cyl}\,\frac{w_{SW}^2}{2}\,A\frac{D}{4} = \frac{K}{\mathrm{Re}^n}\,\frac{2\pi R_b\left(D+h_b+x\right)}{4}\,\rho_{cyl}\,\frac{\left(R_b\omega_b\right)^2}{2}\,\frac{D}{4} =$$

$$= \frac{K}{\mathrm{Re}^n}\,\frac{\pi\left(D+h_b+x\right)}{2}\,\rho_{cyl}\,\omega_b^2 R_b^3\,\frac{D}{8}$$

$$\zeta_{SW} = \frac{K}{\mathrm{Re}^n}\,\frac{\pi R_b\left(D+h_b+x\right)}{2\left(R_b h_b + \frac{Dx}{2}\right)}$$

Moment of inertia

$$I_{SW} = \frac{\pi\rho_{cyl}}{2}\left(z\,\frac{D^2}{4}\,\frac{D^2}{4} + h_b\,\frac{D_b^4}{16}\right) = \frac{\pi}{32}\,\rho_{cyl}\left(xD^4 + h_b D_b^4\right)$$

$$\frac{dI_{SW}}{dt} = \frac{\pi}{32}\left(xD^4 + h_b D_b^4\right)\frac{d\rho_{cyl}}{dt} + \frac{\pi}{32}\,\rho_{cyl}\,D^4\,\frac{dx}{dt} =$$

$$= \frac{\pi}{32}\left(xD^4 + h_b D_b^4\right)\left(\frac{\dot{m}_{IO}}{V_{cyl}} - \frac{A_P\,\rho_{cyl}}{V_{cyl}}\,\frac{dx}{dt}\right) + \frac{\pi}{32}\,\rho_{cyl}\,D^4\,\frac{dx}{dt}$$

For energy balance

$$\omega_{SW} I_{SW} \frac{d\omega_{SW}}{dt} = \omega_{SW} SWN \frac{\pi n_M}{30} \frac{1}{8} D^2 \dot{m}_{IO} - \frac{\zeta_{SW}}{2} \omega_{SW}^3 R_b^2 \rho_{cyl} \frac{D\omega_{SW}}{4} \left(R_b h_b + \frac{Dx}{2} \right) - \omega_{SW}^2 \frac{dI_{SW}}{dt}$$

$$\frac{dK_{in,SW,IO}}{dt} = \omega_{SW} SWN \frac{\pi n_M}{30} \frac{1}{8} D^2 \dot{m}_{IO} - \frac{\zeta_{SW}}{8} \omega_{SW}^3 R_b^2 D\rho_{cyl} \left(R_b h_b + \frac{Dx}{2} \right) - \frac{\omega_{SW}^2}{2} \frac{dI_{SW}}{dt}$$

$$\frac{dK_{PIST,IO}}{dt} = \dot{m}_{IO} \frac{w_{IO}{}^2}{2}$$

$$\frac{dK_{PIST,SW}}{dt} = \pi \frac{\omega_{SW}^2 \rho_{cyl}}{4} \frac{D^4}{16} \frac{dx}{dt}$$

$$g_{SW,IO} = \dot{m}_{IO} \frac{w_{IO}{}^2}{2} + \pi \frac{\omega_{SW}^2 \rho_{cyl}}{4} \frac{D^4}{16} \frac{dx}{dt} - \omega_{SW} SWN \frac{\pi n_M}{30} \frac{1}{8} D^2 \dot{m}_{IO} +$$

$$+ \frac{\zeta_{SW}}{8} \omega_{SW}^3 R_b^2 D\rho_{cyl} \left(R_b h_b + \frac{Dx}{2} \right) + \frac{\omega_{SW}^2}{2} \frac{dI_{SW}}{dt}$$

During compression, swirl speed may increase due to the reduction of swirl moment of inertia (diameter) if piston bowl is used and decelerates due to density increase and loss caused by friction torque. Assuming the averaged swirl angular speed for the system of bowl and cylinder diameters, the equations for swirl angular momentum and kinetic energy yield

$$\omega_{SW} I_{SW} \frac{d\omega_{SW}}{dt} = -\frac{\zeta_{SW}}{8} \omega_{SW}^3 R_b^2 D\rho_{cyl} \left(R_b h_b + \frac{Dx}{2} \right) - \omega_{SW}{}^2 \frac{dI_{SW}}{dt}$$

$$\frac{dK_{in,SW,C}}{dt} = -\frac{\zeta_{SW}}{8} \omega_{SW}^3 R_b^2 D\rho_{cyl} \left(R_b h_b + \frac{Dx}{2} \right) - \frac{\omega_{SW}^2}{2} \frac{dI_{SW}}{dt}$$

$$\frac{dK_{PIST,SW,C}}{dt} = \pi \frac{\omega_{SW}^2 \rho_{cyl}}{4} \frac{\pi D^4}{16} \frac{dx}{dt}$$

$$g_{SW,C} = \frac{dK_{PIST,SW,C}}{dt} + \frac{\zeta_{SW}}{8} \omega_{SW}^3 R_b^2 \rho_{cyl} \frac{R_{SW}}{2} \left(R_b h_b + \frac{Dx}{2} \right) + \frac{\omega_{SW}^2}{2} \frac{dI_{SW}}{dt}$$

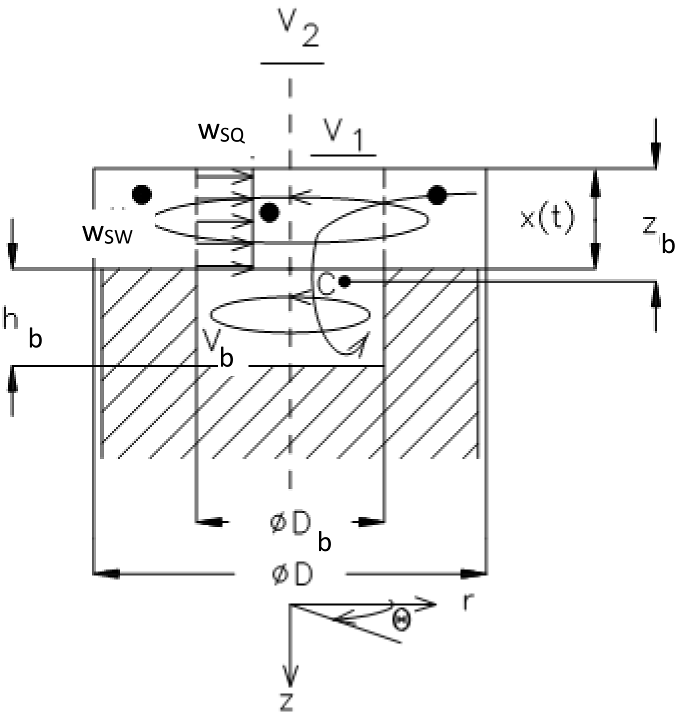

Figure 2 Swirl combined with squish restriction of outer diameter in a piston bowl

Longitudinal Motion of Piston

Kinetic energy of flow, caused by piston longitudinal motion, can be integrated using approximately triangular distribution of velocity between a piston crown and cylinder head - Figure 3. It can be added to swirl motion only (perpendicular components of velocity), the combination with squish or tumble is possible only if the difference between both energies is large (more than one order of magnitude).

$$w_P = \frac{dx}{dt}$$

$$K_L = \frac{\rho_{cyl} V_b}{2} w_P^2 + \frac{\rho_{cyl}}{2} \int_0^x w_P^2 \left(\frac{\xi}{x}\right)^2 A_P d\xi = \frac{\rho_{cyl} V_b}{2} w_P^2 \left(1 + \frac{A_P}{V_b} \int_0^x \left(\frac{\xi}{x}\right)^2 d\xi\right) =$$

$$= \frac{\rho_{cyl} V_b}{2} w_P^2 \left(1 + \frac{A_P}{3V_b} x\right) = \frac{\rho_{cyl} V_b}{2} \left(\frac{dx}{dt}\right)^2 \left(1 + \frac{A_P}{3V_b} x\right)$$

$$\frac{dK_L}{dt} = \frac{V_b}{2} \left(\frac{dx}{dt}\right)^2 \left(1 + \frac{A_P}{3V_b} x\right) \frac{d\rho_{cyl}}{dt} + \rho_{cyl} V_b \left(1 + \frac{A_P}{3V_b} x\right) \frac{dx}{dt} \frac{d^2 x}{dt^2} + \frac{\rho_{cyl} V_b}{2} \frac{A_P}{3V_b} \left(\frac{dx}{dt}\right)^3$$

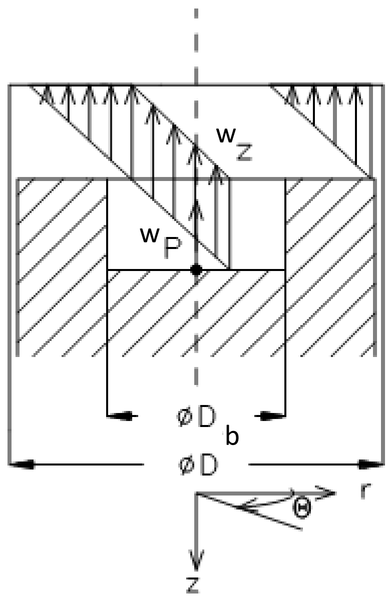

Figure 3 Longitudinal motion in a cylinder ´

In this case, there is difficult to distinguish between bulk flow kinetic energy generators and sinks. The second term, causing oscillations, is most probably the immediate source of turbulence, other two terms are piston velocity generators.

Tumble

Tumble motion cannot be combined with squish (which is of the same substance but induced by different methods) but it can be combined with swirl. The equations are independent in such a case since both velocities are mutually orthogonal and kinetic energies may be simply added.

Simplified tumble surrogate has been developer using two hemi-spherical vortexes connected by cylindrical rod vortex if tumble radius is less than that of cylinder. In extreme case of a vortex developed over the whole cylinder width, sphere vortex occurs. During compression, tumble vortex is reduced in size due to interaction with a cylinder head and a piston. Pent-roof combustion chambre features an angle of side plane from plane of piston bottom β, piston hemi-spherical cavity of depth ***b*** and radius ***R_b***, distance between piston crown periphery and the lowest plane of cylinder head bottom ***x.*** Radius of hemi-spherical vortex is determined by relation

$$x + b + \frac{D \operatorname{tg}\beta}{2} = R_{TU} + R_{TU}\cos\beta + R_{TU}\sin\beta\operatorname{tg}\beta$$

$$R_{TU} = \min\left(\frac{D}{2}; \frac{x + b + \frac{D}{2}\operatorname{tg}\beta}{1 + \frac{1}{\cos\beta}}\right)$$

$$\frac{dR_{TU}}{dt} = \text{if } R_{TU} < \frac{D}{2} \text{ then } \frac{1}{1 + \frac{1}{\cos\beta}}\frac{dx}{dt} \text{ else } 0$$

Cylinder volume with hemispherical bowl in a piston and pent-roof space in a cylinder head is

$$V_{cyl} = \frac{\pi D^2}{4}x + \frac{\pi D^2}{4}\left(h_{Pi,HLb} + \frac{D}{4}\operatorname{tg}\beta\right) + \frac{\pi b}{6}\left(3R_{du}^2 + b^2\right)$$

or (preferably)

$$V_{cyl} = \frac{\pi D^2}{4}x + \frac{\pi D^2}{4}\left(h_{Pi,HL} + \frac{D}{4}\operatorname{tg}\beta\right) + \frac{\pi b^2}{3}\left(3R_b - b\right)$$

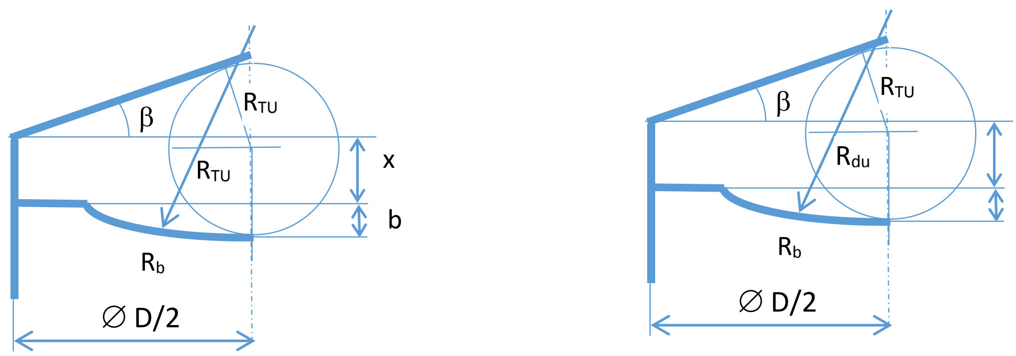

Figure 4 Tumble vortex dimensions in a pentroof combustion chamber

Moreover, if b>0 then $R_b = \frac{D}{2}$ (formula with R_{du} cannot be used) and otherwise

$$R_{du} = \frac{b}{2} + \frac{R_b^2}{2b}$$

Tumble vortex includes generally two hemispherical parts of the height of h_{HS} and radius of $R_{TU} < D/2$ and cylindrical part with the same radius and length of $h_{TU} = D - 2h_{HS}$. The length is

$$h_{TU} = \sqrt{D^2 - 4R_{TU}^2}$$

$$h_{HS} = \frac{D - \sqrt{D^2 - 4R_{TU}^2}}{2}$$

$$\frac{dh_{HS}}{dt} = \frac{2R_{TU}}{\sqrt{D^2 - 4R_{TU}^2}} \frac{dR_{TU}}{dt}$$

Volume of a vortex and its time derivative are

$$V_{TU} = \pi R_{TU}^2 \sqrt{D^2 - 4R_{TU}^2} + \frac{2\pi}{3} h_{Hs}^2 \left(3R_{TU} - h_{Hs}\right)$$

$$\frac{dV_{TU}}{dt} = \left(2\pi R_{TU} \sqrt{D^2 - 4R_{TU}^2} - \frac{4\pi R_{TU}^3}{\sqrt{D^2 - 4R_{TU}^2}}\right) \frac{dR_{TU}}{dt} +$$

$$+ \frac{4\pi}{3} h_{Hs} \left(3R_{TU} - h_{Hs}\right) \frac{dh_{Hs}}{dt} + \frac{2\pi}{3} h_{Hs}^2 \left(3\frac{dR_{TU}}{dt} - \frac{dh_{Hs}}{dt}\right)$$

Moment of inertia of a hemispherical part of the height of h_{HS} is

$$I_{HS} = \frac{\pi \rho_{cyl}}{2} \int_{R_{TU} - h_{HS}}^{R_{TU}} \left(R_{TU}^2 - y^2\right)^2 dy =$$

$$= \frac{\pi \rho_{cyl}}{30} \left(8R_{TU}^5 - 15R_{TU}^4 \left(R_{TU} - h_{HS}\right) + 10R_{TU}^2 \left(R_{TU} - h_{HS}\right)^3 - 3\left(R_{TUY} - h_{HS}\right)^5\right)$$

$$I_S = 2\frac{\pi \rho_{cyl}}{30} 8R_S^5 = \frac{8\pi \rho_{cyl}}{15} R_S^5 = \frac{4\pi \rho_{cyl}}{3} R_S^3 \frac{2}{5} R_{TU}^2 = \frac{2}{5} m R_S^2$$

As an extreme, it yields sphere moment of inertia $\boldsymbol{I_S}$.

Moment of inertia of tumble vortex and its time derivative are

$$I_{TU} = 2I_{HS} + \frac{\pi \rho_{cyl}}{2} R_{TU}^4 \sqrt{D^2 - 4R_{TU}^2}$$

$$\frac{dI_{HS}}{dt} = \frac{\pi}{30}\left(8R_{TU}^5 - 15R_{TU}^4\left(R_{TU} - h_{HS}\right) + 10R_{TU}^2\left(R_{TU} - h_{HS}\right)^3 - 3\left(R_{TU} - h_{HS}\right)^5\right)\frac{d\rho_{cyl}}{dt} +$$
$$+\frac{\pi}{30}\rho_{cyl}\frac{dR_{TU}}{dt}\left[40R_{TU}^4 - 60R_{TU}^3\left(R_{TU} - h_{HS}\right) + 20R_{TU}\left(R_{TU} - h_{HS}\right)^3\right] +$$
$$+\frac{\pi}{30}\rho_{cyl}\left(\frac{dR_{TU}}{dt} - \frac{dh_{HS}}{dt}\right)\left[-15R_{TU}^4 + 30R_{TU}^2\left(R_{TU} - h_{HS}\right)^2 - 15\left(R_{TU} - h_{HS}\right)^4\right]$$

$$\frac{dI_{TU}}{dt} = 2\frac{dI_{HS}}{dt} + 2\pi\rho_{cyl}R_{TU}^3\sqrt{D^2 - 4R_{TU}^2}\frac{dR_{TU}}{dt} - 2\pi\rho_{cyl}R_{TU}^5\frac{1}{\sqrt{D^2 - 4R_{TU}^2}}\frac{dR_{TU}}{dt} + \frac{\pi}{2}R_{TU}^4\sqrt{D^2 - 4R_{TU}^2}\frac{d\rho_{cyl}}{dt}$$

$$\frac{d\rho_{cyl}}{dt} = \frac{d\frac{m}{V}}{dt} = \frac{\frac{dm}{dt}V - m\frac{dV}{dt}}{V^2} = \frac{1}{V_{cyl}}\left(\frac{dm_{cyl}}{dt} - \rho_{cyl}A_{pist}\frac{dx}{dt}\right)$$

Angular speed of tumble can be integrated during inlet stroke adding contributions of tumble flow through inlet valve. During a steady flow test, tumble contribution is characterized by a tumble number ***TUN***, calculated from equivalent engine speed at steady flow through half of cylinder cross/section, i.e., from equivalent mean piston speed:

$$dI_{test} = \frac{\pi\rho_{cyl}D^2}{4}\frac{\dot{m}_{IO}}{\frac{\pi D^2}{8}\rho_{cyl}}dt\frac{2D^2}{5} = \frac{4\pi}{5}\rho_{cyl}D^4\frac{Zn_M}{30}dt$$

$$\frac{dB_{test}}{dt} = \omega_{test}\frac{dI_{test}}{dt} = TUN\frac{\pi n_M}{30}\frac{4\pi}{5}\rho_{cyl}ZD^4\frac{n_M}{30} = TUN\frac{4\pi^2}{5}\rho_{cyl}ZD^4\frac{n_M^2}{900}$$

During inlet stroke, the time-rate of inlet flow contribution to angular momentum in cylinder is

$$\frac{dB_{IT}}{dt} = \dot{m}_{IO}\,\overline{w_{IO,t,SW}\frac{R_{TU}}{2}} = TUN\frac{\pi n_M}{60}R_{TU}{}^2\dot{m}_{IO}$$

whereas ***TUN*** and mass flow rate are functions of valve stroke and piston position. They may be related to averaged tumble radius. As in the case of swirl, the mean diameter of vortex is assumed for angular momentum addition.

Swirl or tumble creates pressure fields due to centrifugal force in a cylinder, which influences piston power by work $\frac{dx}{dt}\sum A_P\Delta p_C$ (piston consumes it during inlet as positive, delivers it during compression as negative, etc.), The contributions to pressure field caused by quadrates of mutually perpendicular angular speeds may be added but integrated in different limits (swirl up to cylinder diameter, tumble inside dimensions of tumble vortex only). Generally

$$dF_C = (\omega_{SW}^2 + \omega_{TU}^2) r\,\rho_{cyl}\, zrd\varphi dr$$

$$pzrd\varphi + pd\varphi\, zdr - (p + dp)z(r + dr)d\varphi + dF_C = 0$$

$$-rdp + \omega_{tot}^2 r\,\rho_{cyl}\, rdr = 0 \Rightarrow \frac{dp}{dr} = (\omega_{SW}^2 + \omega_{TU}^2) r\,\rho_{cyl}$$

$$\Delta p_{C,TU} = \omega_{TU}^2 \rho_{cyl} \frac{r^2}{2}$$

$$\Delta F_{C,TU} = \int_0^{D/2} \Delta p\; 2\pi r dr = \pi \frac{\omega_{TU}^2 \rho_{cyl}}{4} R_{TU}^4$$

Then, using friction torque force at tumble radius. Friction is estimated from a "pipe" flow of half tumble cross-section with longitudinal length of tumble perimeter and wetted perimeter equal to tumble one

$$M_{TU,loss} = \frac{K}{\mathrm{Re}^n} \frac{L_{long} O}{4A} \rho_{cyl} \frac{w_{TU}^2}{2} AR_{TU} = \frac{K}{\mathrm{Re}^n} \frac{2\pi R_{TU} (\pi R_{TU} + h_{TU})}{4} \rho_{cyl} \frac{(R_{TU}\omega_{TU})^2}{2} R_{TU} =$$

$$= \frac{K}{4\,\mathrm{Re}^n} \pi(\pi R_{TU} + h_{TU})\rho_{cyl}\omega_{TU}^2 R_{TU}^4$$

$$\zeta_{TU} = \frac{K}{\mathrm{Re}^n} \frac{2\pi R_{TU}(\pi R_{TU} + h_{TU})}{R_{TU}h_{TU} + 2R_{TU}^2 \frac{1}{2}\arcsin\frac{2R_{TU}}{D} - 2\frac{1}{4}R_{TU}h_{TU}} = \frac{K}{\mathrm{Re}^n} \frac{2\pi R_{TU}(\pi R_{TU} + h_{TU})}{\frac{R_{TU}h_{TU}}{2} + R_{TU}^2 \arcsin\frac{2R_{TU}}{D}}$$

Angular momentum conservation has to take into account moment of inertia change due to change of vortex mass and gyration radius, momentum convection during inlet stroke and momentum convection due to change of mass inside a vortex. The change of mass is caused by volume and density changes. The change of volume is not caused by re-shaping the tumble vortex, as it is in the case of swirl being compressed into a piston bowl due to overall change of density, but by convection of angular momentum to or from a vortex; in the case of transfer of surrounding gas into a vortex no vorticity is assumed, if transfer from a vortex occurs, the angular momentum is lost (i.e., transformed to turbulent kinetic energy)

$$\frac{dm_{TU,C}}{dt} = \rho_{cyl} \frac{dV_{TU}}{dt} + V_{TU} \frac{d\rho_{cyl}}{dt}$$

$$\text{if } \frac{dm_{TU,C}}{dt} > 0 \text{ then } \frac{dB_C}{dt} = 0 \text{ else}$$

$$\frac{dB_C}{dt} = \frac{B_{TU}}{V_{TU}\rho_{cyl}} \left(\rho_{cyl} \frac{dV_{TU}}{dt} + V_{TU} \frac{d\rho_{cyl}}{dt} \right)$$

$$\frac{dB_{TU}}{dt} = I_{TU} \frac{d\omega_{TU}}{dt} + \omega_{TU} \frac{dI_{TU}}{dt} = \frac{dB_{IT}}{dt} + \frac{dB_C}{dt} - \frac{K}{4\,\mathrm{Re}^n} \pi(\pi R_{TU} + h_{TU})\rho_{cyl}\omega_{TU}^2 R_{TU}^4$$

$$\frac{d\omega_{TU}}{dt} = \frac{1}{I_{TU}} \left(\frac{dB_{IT}}{dt} + \frac{dB_C}{dt} - \frac{K}{4\,\mathrm{Re}^n} \pi(\pi R_{TU} + h_{TU})\rho_{cyl}\omega_{TU}^2 R_{TU}^4 - \omega_{TU} \frac{dI_{TU}}{dt} \right)$$

For energy balance it is assumed that the piston work against pressure difference at inlet valve has been already taken into account. Otherwise, the procedure is similar to that of swirl kinetic energy and its decay simulation

$$\omega_{TU} I_{TU} \frac{d\omega_{TU}}{dt} = \omega_{TU} \frac{dB_{IT}}{dt} + \omega_{TU} \frac{dB_C}{dt} - \frac{K}{4\,\mathrm{Re}^n} \pi\left(\pi R_{TU} + h_{TU}\right)\rho_{cyl}\,\omega_{TU}^3 R_{TU}^4 - \omega_{TU}^2 \frac{dI_{TU}}{dt}$$

$$\frac{dK_{in,TU}}{dt} = \omega_{TU} \frac{dB_{IT}}{dt} + \omega_{TU} \frac{dB_C}{dt} - \frac{K}{4\,\mathrm{Re}^n} \pi\left(\pi R_{TU} + h_{TU}\right)\rho_{cyl}\,\omega_{TU}^3 R_{TU}^4 - \frac{\omega_{TU}^2}{2} \frac{dI_{TU}}{dt}$$

$$\frac{dK_{PIST,TU}}{dt} = \pi \frac{\omega_{TU}^2 \rho_{cyl}}{4} R_{TU}^4 \frac{dx}{dt}$$

$$g_{TU} = \frac{dK_{PIST,TU}}{dt} - \omega_{TU} \frac{dB_{IT}}{dt} - \omega_{TU} \frac{dB_C}{dt} +$$

$$+ \frac{K}{4\,\mathrm{Re}^n} \pi\left(\pi R_{TU} + h_{TU}\right)\rho_{cyl}\,\omega_{TU}^3 R_{TU}^4 + \frac{\omega_{TU}^2}{2} \frac{dI_{TU}}{dt}$$

The effect of kinetic energy in valve seat, delivered by a piston, is fully respected for swirl motion from port integral features. That is why it is not repeated here. Effect of inlet mass flow is added only if the mass flow rate is positive.

During compression, tumble speed may increase due to the reduction of tumble moment of inertia and decelerates due to density increase and loss caused by friction torque. The equations for tumble angular momentum and kinetic energy are the same as during inlet process but with zero mass flow rate.

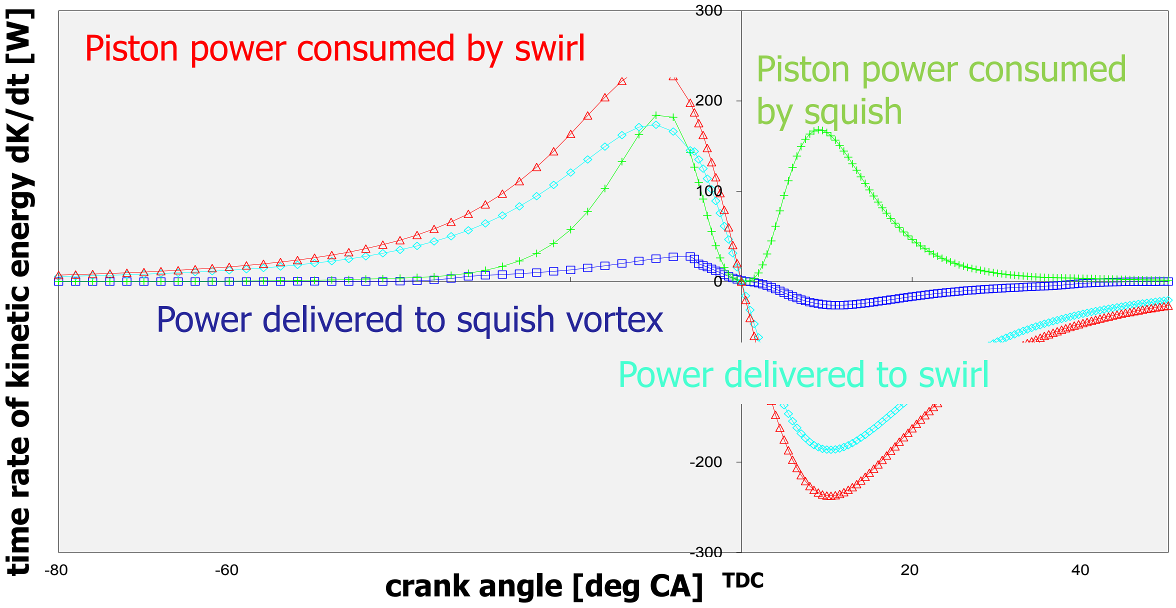

Figure 5 Example of bulk vortex power for engine of D/Z 120/140 mm (piston bowl 50/40 mm), compression ratio 17, 2 000 rpm

Conclusion

Main issue of full CFD models is lack of sufficiently flexible calibration parameters before fully predictive turbulent transport models are developed. Simulations have to be calibrated by spatially averaged data or even using steady-flow measurements at physical models only.

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