

---

## Predictive control of commercial e-vehicle using a priori route information

---

Pavel Steinbauer\* and Josef Husák

Faculty of Mechanical Engineering,  
Czech Technical University in Prague,  
Prague, Czech Republic  
Email: pavel.steinbauer@fs.cvut.cz  
Email: josef.husak@fs.cvut.cz

\*Corresponding author

Florent Pasteur

Siemens Industry Software S.A.S. DF PL STS CAE 1D,  
Lyon, France  
Email: florent.pasteur@siemens.com

Petr Denk, Jan Macek and Zbynek Šika

Faculty of Mechanical Engineering,  
Czech Technical University in Prague,  
Prague, Czech Republic  
Email: petr.denk@fs.cvut.cz  
Email: jan.macek@fs.cvut.cz  
Email: zbynek.sika@fs.cvut.cz

**Abstract:** The driving range of the vehicle is usually an issue due to the limited energy storage capacity of the acu-pack. Thus, the e-vehicle control towards energy consumption decrease is of extreme importance. The known information about route properties can be used to plan torque/braking profile in an optimal way. Several approaches are compared. The first is design approach based on model predictive control (MPC) in combination with prior (before the trip starts) dynamic optimisation, the other is model-predictive control using hard limits based on route shape analyses and legal limits. The classical, optimised PID control is used as reference driver. A detailed driving range estimation model of a Fiat Doblo e-vehicle is the basis, including the main e-vehicle subsystem 1D model, e-motor, battery pack, air-conditioning/heating and EVCU. The model calibration is based on real vehicle measurements.

**Keywords:** e-vehicle; model predictive control; MPC; range extension; range estimation model.

**Reference** to this paper should be made as follows: Steinbauer, P., Husák, J., Pasteur, F., Denk, P., Macek, J. and Šika, Z. (2018) 'Predictive control of commercial e-vehicle using a priori route information', *Int. J. Powertrains*, Vol. 7, Nos. 1/2/3, pp.53–71.

**Biographical notes:** Pavel Steinbauer is a Research Fellow at the CTU in Prague. He is interested in mechanics, mechatronics, control, vibration and noise.

Josef Husák is a researcher and he is interested in mechatronics and control.

Florent Pasteur is expert in multi-physics dynamic system modelling, especially hybrid and electric vehicles.

Petr Denk is a researcher working in the field of applied mechanics, vehicle dynamics and optimisation.

Jan Macek is a Full Professor at the CTU in Prague. He is interested in thermodynamics, engine modelling, simulation, optimisation and control.

Zbynek Šika is a Full Professor at CTU in Prague. He is interested in mechanics, mechatronics, robotics, truss structure design, optimisation and control, and vibration control.

This paper is a revised and expanded version of a paper entitled ‘Predictive control of commercial e-vehicle using a priori route’ presented at the Powertrain Modelling and Control, Testing, Mapping and Calibration, Loughborough, 7–9 September 2016.

## 1 Introduction

Current vehicles, especially electric ones are being equipped with more and more complex controllable structures and subsystems. It means that control strategy for complex, highly nonlinear dynamic systems must be designed. In addition, a lot of information about planned route is known in advance. Especially slope of the route can be obtained from open resources as well as geographic coordinates of the route. These pieces of information are linked with legal velocity limits, traffic density etc. On the other hand, the amount of energy stored on-board is quite limited in case of e-vehicle. In this paper, control strategies based on detailed simulation model of the vehicle are evaluated.

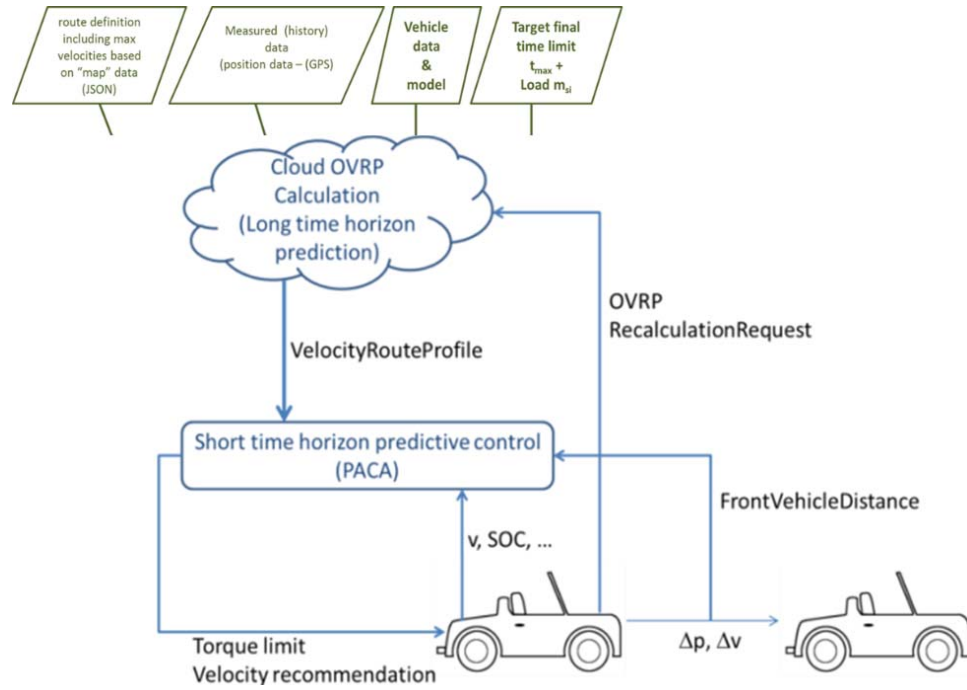
The mechatronic nature of current vehicles together with widely available high capacity data connection and various information resources attract a lot of research effort to increase energy efficiency of vehicle operation. In particular, algorithms to find optimal routing with respect to energy consumption [called eco-routing, e.g., (Fu et al., 2004; Minett et al., 2011; Saboohi and Farzaneh, 2009; Barth and Boriboonsomsin, 2009)] based on 3D map data are being developed. Also, there are eco-driving initiatives which provide drivers with guidelines for efficient behaviour and eco-driving systems studying strategies for efficient driving within traffic with varying congestion.

Presented approach uses prior optimisation of velocity profile and complements it by nonlinear model-based predictive controller, which adapts required velocity profile according to the vehicle and traffic state.

The optimisation is based on segmentation of the route in sections with constant properties. Thus, special semi-integrated, fast numerical solution of a set of nonlinear differential equations can be used based on in-section averaged parameters. Between sections, the values change. Furthermore, backward in-time predictor is used for finding

maximum velocity, state-of-charge (SOC) and heating, ventilating, air-conditioning (HVAC) power envelope, satisfying future corrector forward-going final solution of braking, charging of batteries and HVAC control with heat pumps and heat accumulation in electric equipment cooling system and HVAC system, including car body thermal capacity. Contribution of optimisation potential from Pareto sets is used. New methods for cargo load estimation from available acceleration and motor power signals are developed.

**Figure 1** Control hierarchy structure (see online version for colours)



## 2 Vehicle model design and verification

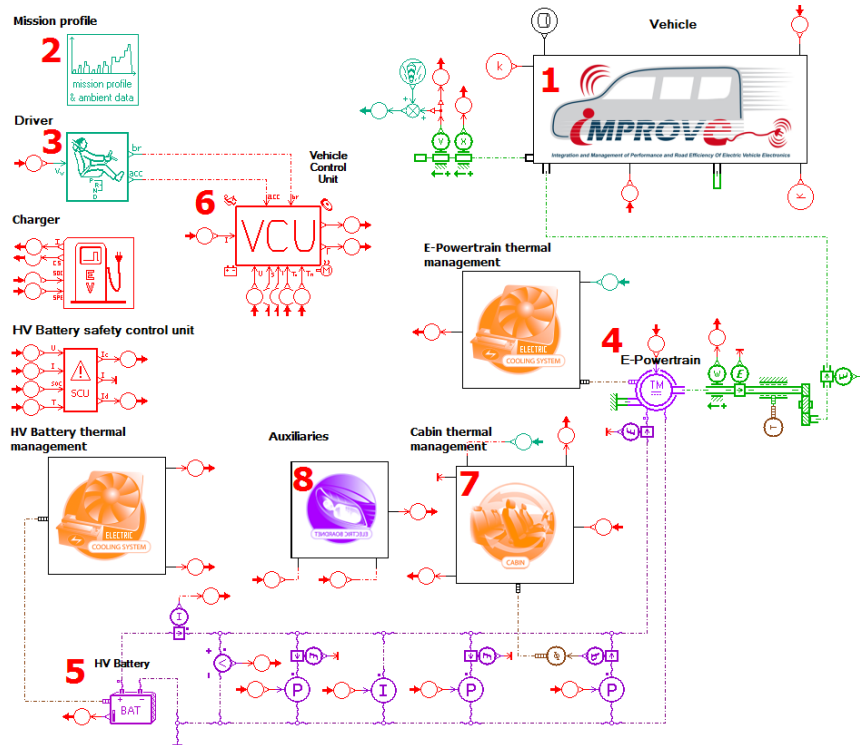
The control design is carried out before the vehicle is available for application. Furthermore, the verification and design experiments would be quite costly on the real vehicle. So development approach design-by-simulation and software in the loop is used.

The vehicle energy management (VEM) simulator is a LMS.IMAGINE.Lab Amesim 1D virtual model of the Fiat Doblo electric vehicle. This simulation platform is well suited to carry on global vehicle energy consumption evaluation (Badin et al., 2015; Maroteaux et al., 2015). It is a driving range prediction model, containing main subsystems: vehicle dynamics 1D model, e-motor, battery pack, air-conditioning/heating and control unit. In this paper, the VEM model features:

- 1 A 1D vehicle model. Both front and rear axles are modelled. It takes into account wind speed, road slope, variable load, braking and driving torque.

- 2 A mission profile. It defines the vehicle linear speed profile as a function of time. It includes also wind speed and road slop definition.
- 3 A driver model. It is based on PID control block to respect the vehicle velocity mission profile.
- 4 An electric powertrain. It is an average model of an electric machine and DC converter using data-files. The losses, the minimum and the maximum torques are defined with data-files of the real machine measurements. It takes into account motor speed and battery maximum battery voltage limit.
- 5 A high voltage battery. It is a quasi-static equivalent electric circuit model. The model parametres are defined with the datasheet measure? Information. It takes into account the open circuit voltage (OCV) and resistance dependent on a function of SOC and temperature. A simplified aging model is also included.
- 6 A vehicle control unit (VCU). It computes the motor torque demand according to vehicle state with respect to a pedal position, battery SOC, inverter power limitation. High frequency control dynamics such as ESP have been neglected.
- 7 The HVAC system is defined as a simplified reduced model. Equivalent cooling and heating performance have been fitted according to a detailed model results.
- 8 The vehicle auxiliaries, i.e., all the vehicle electric equipment that belongs to the low voltage on board network of the vehicle (12V).

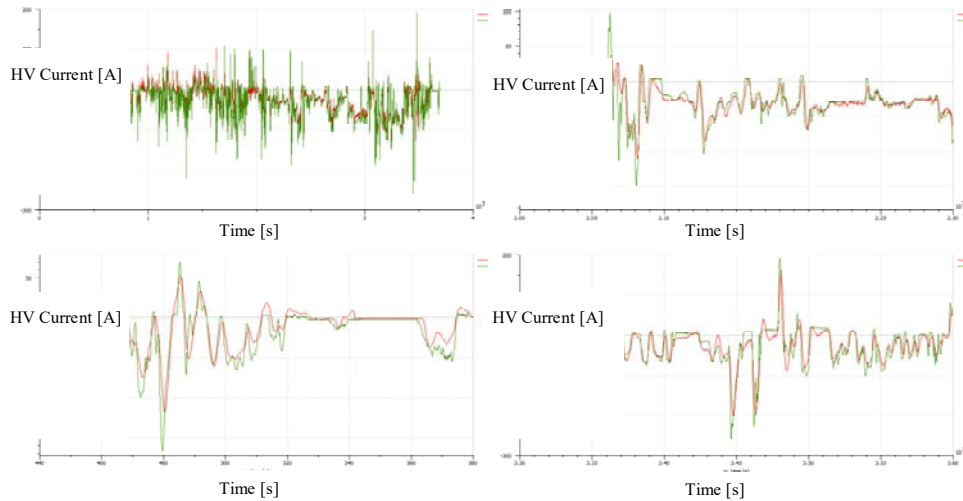
**Figure 2** LMS-Amesim VEM model (see online version for colours)



The operating principle of this simulation is the following. The driver model computes the braking and acceleration pedal signals in order to match the speed profile. The VCU (voltage control unit) converts these commands into a motor torque and mechanical braking command. In that model, there is full regenerative braking torque until reaching the maximum regenerative braking limit. This assumption is justified as the accelerations involved remains small. In practice it is a good approximation for a VEM model. The high voltage battery supplies the electric powertrain and other auxiliaries. In order to achieve a required torque, the electric powertrain model computes the motor current based on a losses table as a function of motor torque, motor speed and battery voltage. Thus, this model assumes a quasi-static variation of torque request compared to the motor natural time constant. This condition is respected in the majority of standard driving situations.

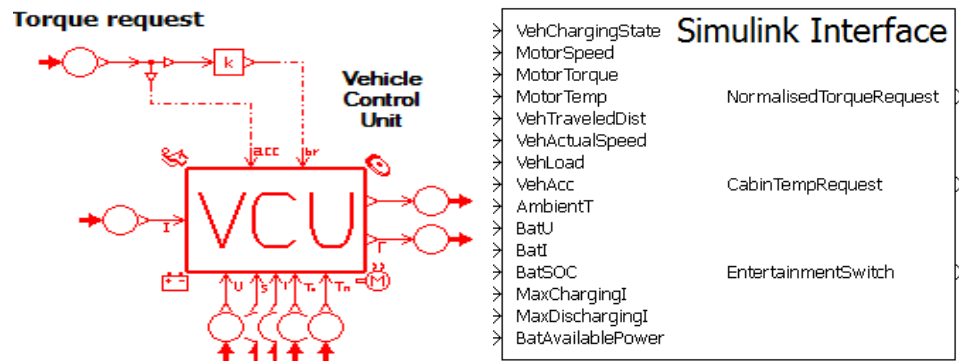
In order to validate this modelling approach, the model has been set up and compared to the baseline vehicle. The vehicle acceleration time has been checked for several vehicle loads. The vehicle energy range and mean consumption have been successfully predicted for the NEDC profile. Additionally, a real velocity profile of the vehicle has been recorded. This data set includes the vehicle CAN bus data time synchronised with the GPS record. This one-hour driving cycle mixes urban and extra urban conditions. In Figure 3, the high voltage battery measured is compared to the model results. A good correlation is achieved between simulation and measurements in all driving situations of the profile. The difference remains acceptable when considering some missing data during the measurement (wind measurements, road slope inaccuracies etc.).

**Figure 3** Comparison of simulation (red) and measure (green) of HV battery current (see online version for colours)



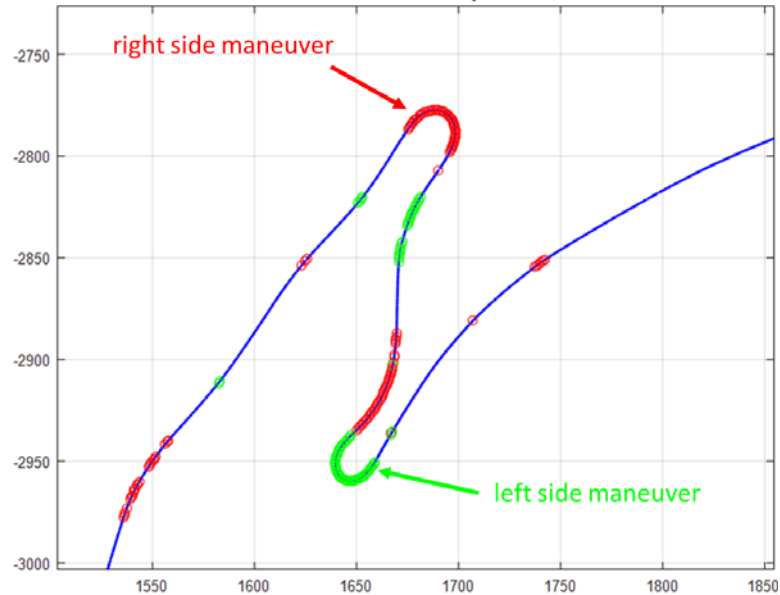
The standalone VEM model has to be modified to be integrated into the off the line predictive controller. A coupling block is added (Figure 4) to define the quantities to be exchanged between LMS-Amesim and Simulink.

The driver model is removed and the Amesim model takes as input the controller torque request.

**Figure 4** Amesim-Simulink coupling block which has driver model disabled (see online version for colours)

### 3 Route dynamic optimisation

Optimal velocity defined along the route can be optimised before the trip starts. It is based on suitable discretisation of the route into sections with constant characteristic properties (maximum achievable velocity, slope, rolling resistance etc.).

**Figure 5** Route section classification based on physical properties of the road shape (see online version for colours)

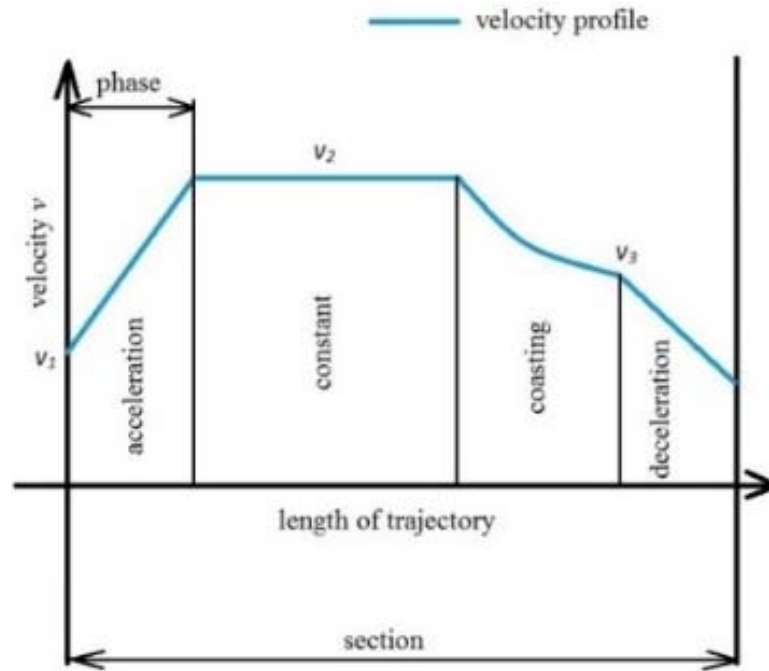
Velocity profile in each section is described by four phases: Acceleration phase with constant acceleration followed by constant velocity phase, then coasting and braking with constant deceleration. Any piecewise velocity profile can be achieved, as the length of any phase can be of zero length. Velocity profile in each section is thus described by five

parameters:  $a_a$  [ $\text{m.s}^{-2}$ ],  $s_a$  [m] – acceleration during acceleration phase and its length,  $a_d$  [ $\text{m.s}^{-2}$ ],  $s_d$  [m] – deceleration during deceleration phase and its length,  $v_1$  [ $\text{m.s}^{-1}$ ] velocity in the beginning of the section (equal to  $v_4$  – output velocity of the previous section),

$$\begin{aligned} v_2 &= \sqrt{2a_a s_a + v_1^2}, & v_3 &= \sqrt{2a_d s_d + v_4^2} \\ s_{coa} &= f(v_2, v_3), & s_{con} &= s - s_a - s_d - s_{coa}(x, u, t) \end{aligned} \quad (1)$$

The velocity profile is optimised using dynamic optimisation (Biegler, 2007; Steinbauer et al., 2016) and provides optimal strategy to achieve savings trading within the given – available travel time.

**Figure 6** Route section velocity description (see online version for colours)



The dynamic optimisation is performed in three steps (Steinbauer et al., 2016). The backward calculation determines maximum velocity envelope, which can be fulfilled based on deceleration capabilities of the vehicle. The forward calculation determines maximum velocity envelope, which can be achieved. Finally, the third step performs global optimisation of all route sections with their parameters.

It is optimised by dynamic system optimisation methods, which were developed for optimisation of complex and nonlinear dynamic processes. They are based on discretisation of the state trajectory and replacement of highly nonlinear dynamic model by algebraic functions.

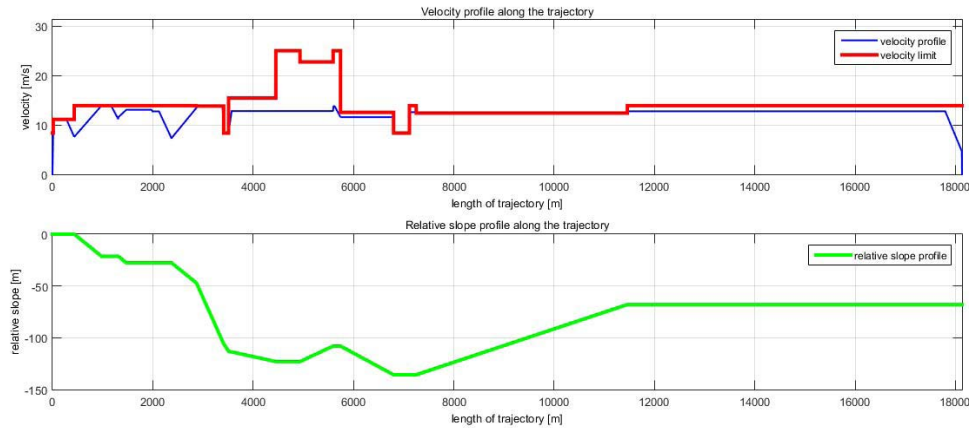
The optimisation criterion (cost function) is overall consumed energy for the whole route. It is calculated by simplified energy consumption relations (mainly algebraic) for each section. The objective function is based on sum of used energies in each section:

- rolling resistance –  $E_f$

- slope resistance –  $E_s$
- acceleration resistance –  $E_a$
- air resistance –  $E_v$
- additional unit of vehicle –  $E_t$
- heating/air conditioning –  $E_h$ .

The objective function is a function of quintuplet  $(v_2, a_a, s_a, a_d, s_d)$  of section parametres multiplied by number of section, along the selected trajectory.

**Figure 7** Optimised velocity profile (see online version for colours)



For such defined optimisation problem and the objective function, the interval of values of the objective function is limited by local and global constraints, named ‘optimisation conditions’. The local optimisation conditions are defined for individual sections of the selected trajectory. They limit the mutual relation of optimisation parametres in each section and prescribed fixed length of each section. Further constraints follow from previous Backward and Forward maximum velocity envelope calculation steps. The minimum velocity envelope must be defined as well, because otherwise to low velocity profile recommendation segments may be calculated which would, in real traffic, be a serious problems because of interaction with faster vehicles in the flow.

Global optimisation conditions define the maximum and minimum values of optimisation parametres, define the continuity of the velocity profile at the endpoint of all sections of the trajectory and the total time of travel of the vehicle along the selected trajectory. All of these conditions form constraint parametre space are to be searched.

Dynamic optimisation of the given problem is based on optimisation technique called, ‘trust-region methods for nonlinear minimisation’. This method is based on the principle of replacing the objective function in a vicinity of an initial estimate by approximation functions. For this approximation function must hold the local and global optimisation conditions.

By finding the minima of this function in the vicinity of the initial estimate the first iteration of the optimisation process is found. This iteration is used as the initial estimate



for second iteration. When the difference of objective function in two consecutive iterations is less than the specified value, the last iteration is considered local minimum. All points of potential minima of the objective function are found initially. The smallest element of this group is the global minimum of the objective function in a prescribed area under consideration.

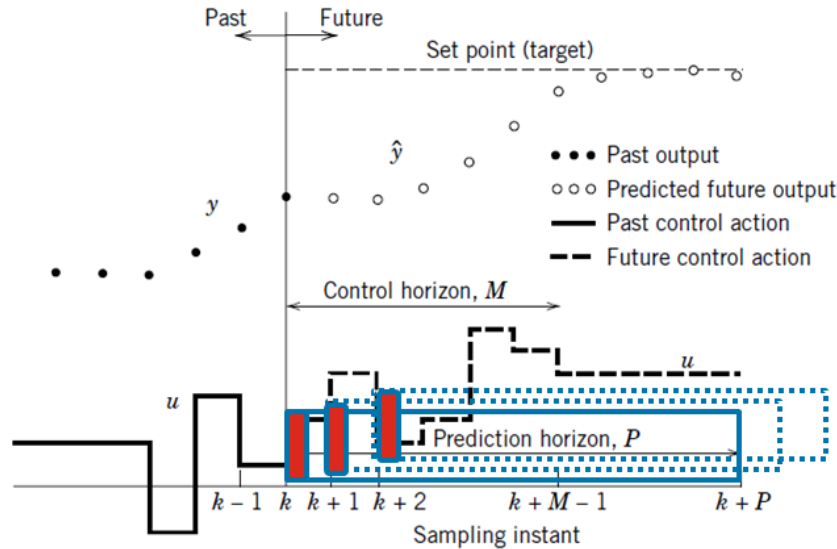
The example of resulting velocity profile shape is shown in Figure 7.

However, the real drive always differs due to the surrounding traffic and vehicle model differences. That is why the on-board real-time controller must be used to adapt the actual torque to achieve optimal behaviour.

#### 4 Closed loop control

The predictive control (model predictive control – MPC) was chosen as a basic tool. It is one of the most advanced methods for control design. This control method was first developed for the petrochemical industry, however, the method become the most widely used advanced technology management throughout the industry. The advantages of this method of regulation are mainly its relative straightforward usage, the ability to control large systems with many inputs and outputs, utilisation of foreseen future requirements on output variables and possibility to include various constraints on control variables, output variables and even their rates (Cannon, 2015).

**Figure 8** Prediction control principle (see online version for colours)



MPC calculations are based on actual measurements of input variables and the prediction of future output values. The aim is to calculate optimal sequence of control actions within the prediction horizon (with respect to selected criteria of optimality) so that the predicted response reached the required value in an optimum manner minimising criterion of optimality defined at prediction horizon.

$$J_k = \sum_{j=1}^N \left( (y_{k+1}^T - w_{k+1}^T) Q_y (y_{k+j} - w_{k+j}) + u_{k+j}^T R_y u_{k+j-1} \right) \quad (2)$$

This resembles open loop control, but only a few first values (control horizon) are used and the calculation is repeated. So, the control loop is closed and the control system can react on deviation of the real system from predicted behaviour (Figure 8).

The MPC approach is straightforward for linear models, so the nonlinear Amesim model.

$$\begin{aligned} \dot{x} &= f(x, u, t) \\ y &= g(x, u) \end{aligned} \quad (3)$$

was linearised using partial derivatives of ‘f’ and ‘g’ funtions according to x and u vectors at operating point into form.

$$\begin{aligned} \Delta \dot{x} &= A \Delta x + B \Delta u \\ y &= C \Delta x + D \Delta u \end{aligned} \quad (4)$$

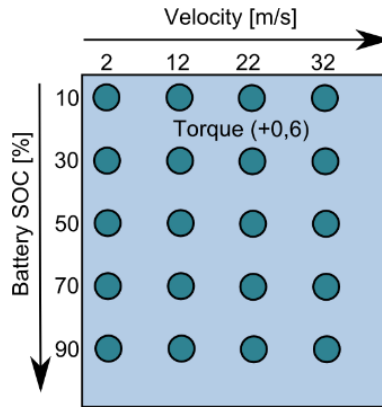
The operating points were selected for system inputs: cabin external temperature difference request [°C], vehicle load [kg], wind speed [m.s<sup>-1</sup>], normalised torque [–], road slope [%] and outputs: Vehicle acceleration [m/s<sup>2</sup>], high battery voltage (HV) [V], HV battery current [A], derivative of battery SOC [s<sup>-1</sup>], vehicle velocity [m.s<sup>-1</sup>].

The continuous linear state space model (4) was discretised and used for linear MPC design, including stability considerations (Appendix I)

$$u_k = e_l (G^T Q G + R)^{-1} G^T Q (-f_l x_k) + e_l (G^T Q G + R)^{-1} G^T Q W \quad (5)$$

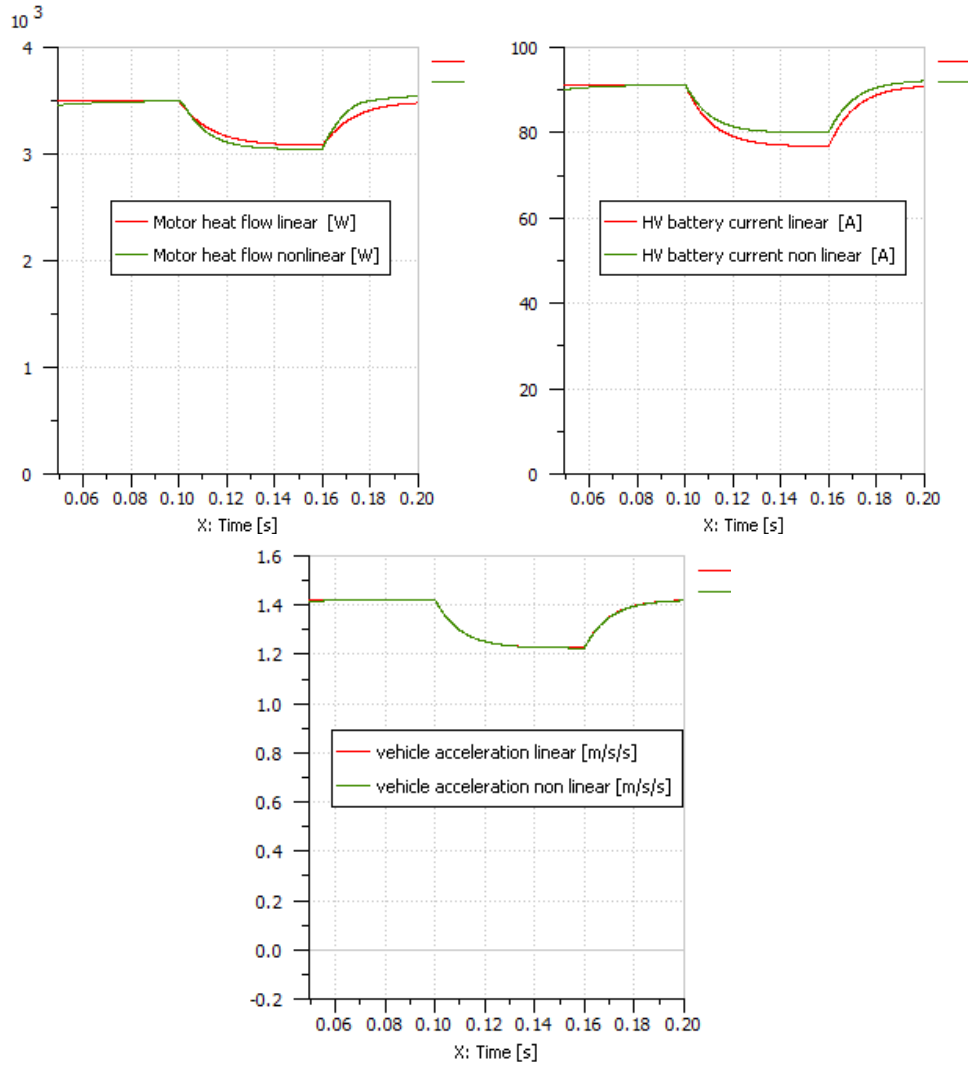
However, the resulting linear controller is valid only close to the linearisation point. To cover nonlinearity of the e-vehicle in the whole range of operating conditions, the linearisation grid of operating points was chosen (Figure 9). For each operating point the linear model was derived and used for control  $u_k = -K_{(v, SOC)} x_k + K_{w(v, SOC)}$  design.

**Figure 9** Linearisation grid (see online version for colours)



In order to check the validity of this approach, each linearised model has been compared with the original nonlinear model. Based on this verification, the linearisation grid density has been adapted to ensure sufficiently accurate results (Figure 9).

**Figure 10** Linear (red) and nonlinear (green) model response to a torque demand decrease at 0.1s (see online version for colours)



These controllers run simultaneously and combined so that suitable output is selected according to the position in the SOC-velocity space. The switching effect is reduced using rate limit for manipulated variable (torque).

The relatively high order linear models are obtained by linearisation routine. The model reduction is thus used to obtain control design model of acceptable size. Unfortunately, the resulting linear model has generally non-zero D matrix. The following method is used to develop model suitable for MPC design procedure. The state-space model is extended by new states based on original system outputs.

$$\dot{\underline{Y}} = \frac{1}{T}(-\underline{Y} + \underline{C}.\underline{X} + \underline{D}.\underline{U}), \quad T \ll \frac{1}{|\text{Re}(\lambda_{\max})|} \quad (6)$$

New output vector 'z' is defined as

$$\underline{Z} = \begin{bmatrix} 0_{yx} & I_{yy} \end{bmatrix} \begin{bmatrix} \underline{X} \\ \underline{Y} \end{bmatrix} \quad (7)$$

It enables to formulate state description with new  $A_n$ ,  $B_n$  and  $C_n$  matrices

$$\begin{bmatrix} \dot{\underline{X}} \\ \dot{\underline{Y}} \end{bmatrix} = \underbrace{\begin{bmatrix} \underline{A} & 0_{xy} \\ \frac{1}{T}\underline{C} & -\frac{1}{T}\underline{I}_{yy} \end{bmatrix}}_{A_n} \begin{bmatrix} \underline{X} \\ \underline{Y} \end{bmatrix} + \underbrace{\begin{bmatrix} \underline{B} \\ \frac{1}{T}\underline{D} \end{bmatrix}}_{B_n} \underline{U} \quad \underline{Z} = \underbrace{\begin{bmatrix} 0_{yx} & I_{yy} \end{bmatrix}}_{C_n} \begin{bmatrix} \underline{X} \\ \underline{Y} \end{bmatrix} \quad (8)$$

For these system matrices the MPC control (Appendix 1) can be used.

The MPC controller design requires selection of many parameters, e.g., controller sampling period, prediction horizon length, control horizon length, maximum change of the required torque (rate). On the other hand, detailed keeping of predefined – selected velocity profile may result in higher energy consumption. These are two conflicting criteria, so Pareto set based on genetic algorithms was utilised to investigate proper settings. Cost functions were deviation from required velocity trajectory

$$D_{MPC} = \int_0^t (v - v_{req})^2 dt \quad (9)$$

And energy consumption

$$E_{MPC} = \int_0^t P_{mot} dt = \int_0^t \omega_{mot} M_{mot} dt = \int_0^t \frac{2\pi n_{mot}}{60} M_{mot} dt \quad (10)$$

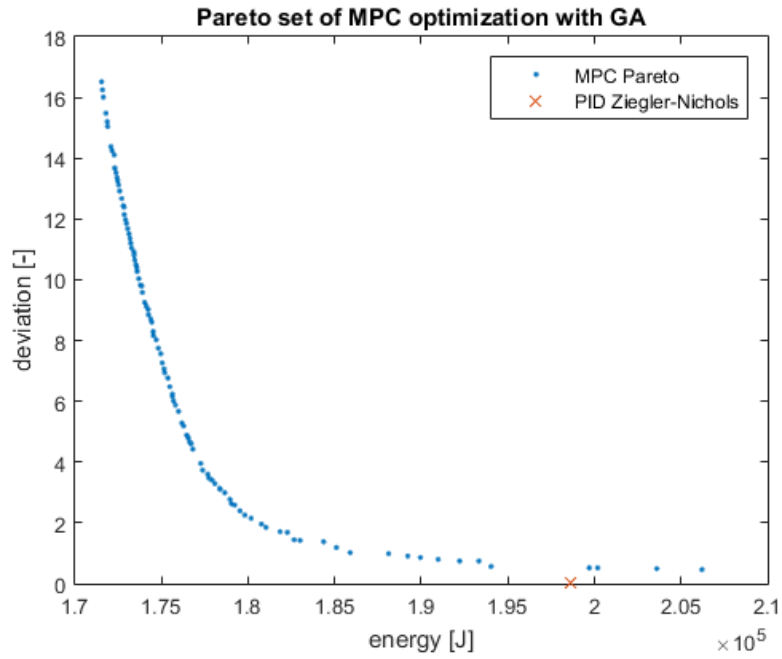
Figure 11 shows resulting Pareto set of possible optimal nonlinear MPC controllers. As reference, PID controller with Ziegler-Nichols settings was selected.

The global stability of closed loop system with such relatively complicated control is difficult to formally establish. However, stability of MPC control of linear system can be ensured, as the control design results in one closed loop gain matrix. In addition, the overall behaviour of the controlled vehicle is checked by simulation experiments.

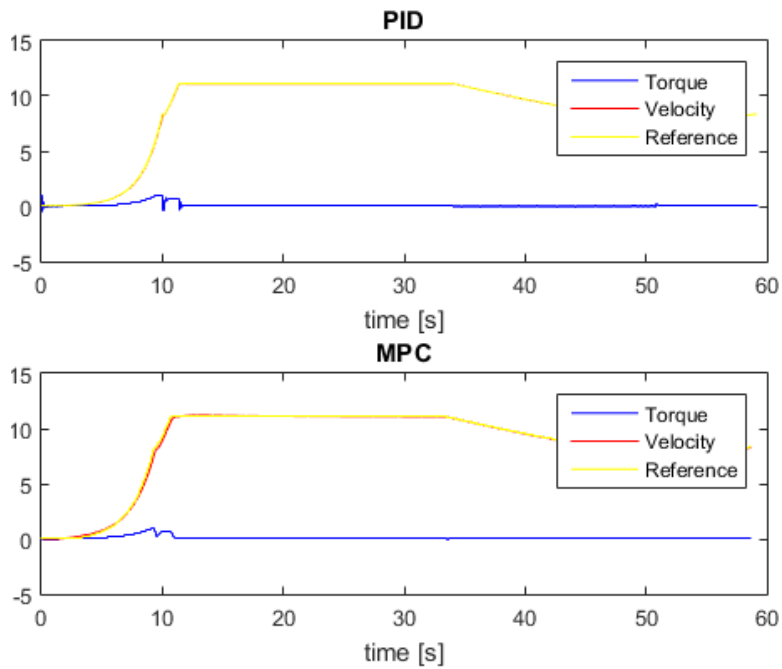
The simulation results show that nonlinear model-based predictive controller may provide superior behaviour in comparison with classical PID control. Not only in terms of energy savings, but also in fluent control actions, which are much more comfortable for passengers (e.g., circled torque at a time of 10s in Figure 12).

Using Pareto multi-criteria optimisation, cost (energy consumption) of keeping predefined velocity profile is investigated and suitable settings of MPC parameters are selected.

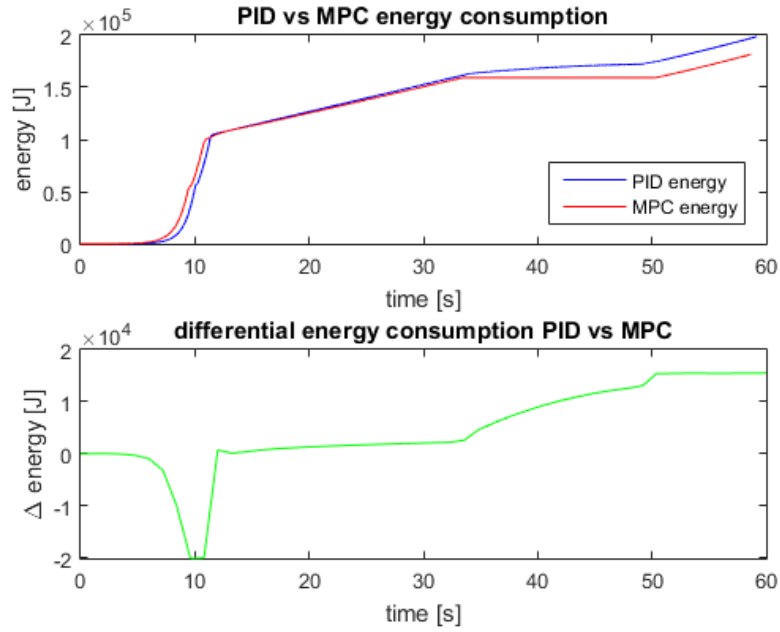
**Figure 11** Pareto set of required velocity trajectory deviation and energy consumption (see online version for colours)



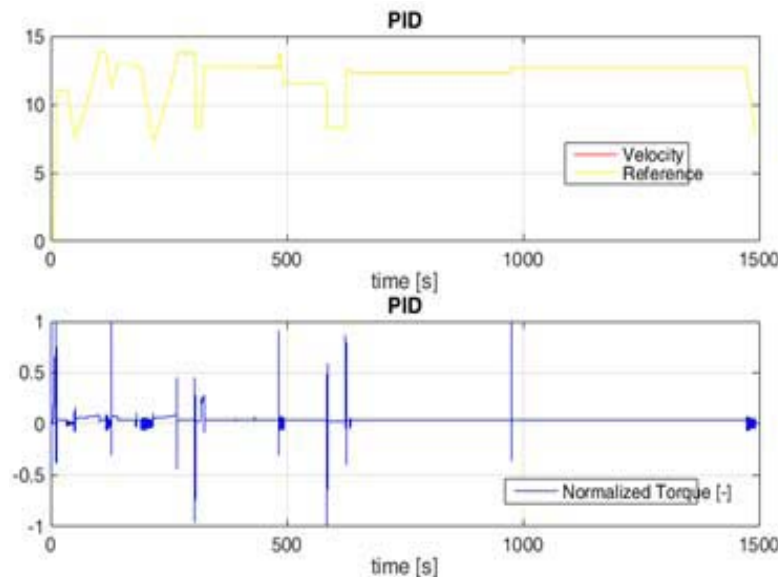
**Figure 12** Demonstration of improved controller behaviour (see online version for colours)



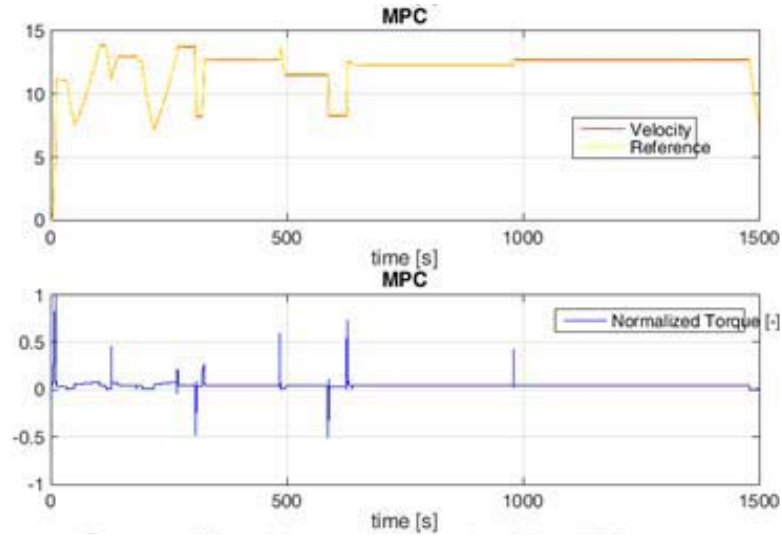
**Figure 13** Demonstration of energy savings on design (short) track (see online version for colours)



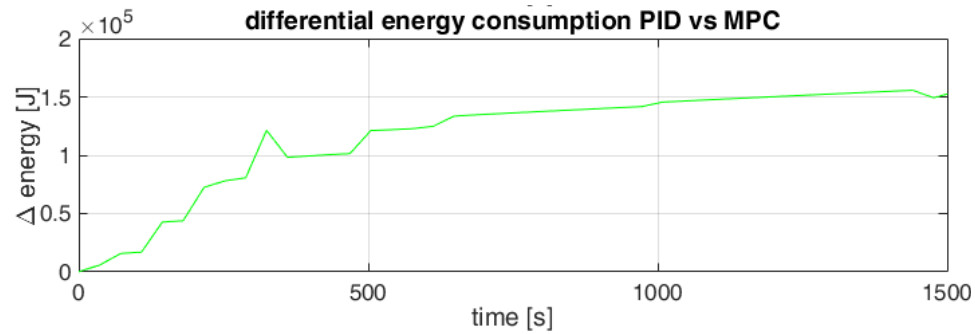
**Figure 14** Control behaviour for reference PID controller and MPC controller (see online version for colours)



**Figure 14** Control behaviour for reference PID controller and MPC controller (continued)  
(see online version for colours)



**Figure 15** Energy consumption comparison (see online version for colours)



In Figure 13 is shown energy saving potential of MPC control on short (design) track, shown in Figure 12. MPC controller ensures smoother control actions leading to lower energy consumption while still maintaining the required velocity profile.

The resulting selected nonlinear MPC controller was tested on reference route of the length of 18 km. Reference control was a discrete PID controller, trying to follow pre-optimised velocity profile. The P, I and D components of the controller were tuned using the Ziegler-Nichols method.

The PID controller exhibits high fluctuation of controlled variable (torque), even switching between acceleration and deceleration (braking) in subsequent control inputs. In contrary, MPC controller exhibits quite smooth control trajectory without braking and the energy consumption is also improved.

In Figure 14 is presented confirmation of energy savings achieved on long route while using nonlinear MPC controller comparing to optimised PID controller.

## 5 Conclusions

The combined control strategy for electric vehicle has been demonstrated, together with preliminary results. The comparison has shown that control design based on accurate MPC provides significant improvements of control behaviour over traditional techniques. The nonlinear control based on MPC gain scheduling and MPC Pareto optimisation demonstrate to be effective tools for control approach for energy savings. The Pareto multi criteria optimisation also supports empirical practical observation that too strict following of pre-optimised velocity profile leads to excessive energy consumption.

## Acknowledgements

This research has been realised using the support of technological agency, Czech Republic, program centres of competence, project # TE01020020 Josef Božek Competence Centre for Automotive Industry and using the support of The Ministry of Education, Youth and Sports program NPU I (LO), project # LO1311 development of vehicle centre of sustainable mobility. This support is gratefully acknowledged.

## References

- Badin, F., Le Berr, F., Castel, G., Dabadie, J.C., Briki, H., Degeilh, P. and Pasquier, M. (2015) 'Energy efficiency evaluation of a plug-in hybrid vehicle under European procedure, worldwide harmonized procedure and actual use', *EVS28 KINTEX*, Korea, 3–6 May 2015.
- Barth, M. and Boriboonsomsin, K. (2009) 'Energy and emissions impacts of a freeway-based dynamic eco-driving system', *Transportation Research Part D: Transport and Environment*, Vol. 14, No. 6, pp.400–410.
- Biegler, L.T. (2007) 'An overview of simultaneous strategies for dynamic optimization', *Chemical Engineering and Processing: Process Intensification*, Vol. 46, No. 11, pp.1043–1053.
- Cannon, M. (2015) *Model Predictive Control* [online] <http://www.eng.ox.ac.uk/~conmrc/mpc/mpc1-2.pdf> (accessed 09 September 2016).
- Fu, M., Li, J. and Deng, Z. (2004) 'A practical route planning algorithm for vehicle navigation system', *Intelligent Control and Automation, 2004. Fifth World Congress on WCICA 2004*, IEEE, Vol. 6.
- Maroteaux, D., Le Guen, D. and Chauvelier, E. (2015) *Development of a Fuel Economy and CO<sub>2</sub> Simulation Platform for Hybrid Electric Vehicles-Application to Renault Eolab Prototype*, SAE Technical Paper, No. 2015-24-2543.
- Minett, C.F. et al. (2001) 'Eco-routing: comparing the fuel consumption of different routes between an origin and destination using field test speed profiles and synthetic speed profiles', in *2011 IEEE Forum on Integrated and Sustainable Transportation System (FISTS)*, IEEE, p.32–39.
- Saboo, Y. and Farzaneh, H. (2009) 'Model for developing an eco-driving strategy of a passenger vehicle based on the least fuel consumption', *Applied Energy*, Vol. 86, No. 10, pp.1925–1932.
- Steinbauer, P. et al. (2016) *Dynamic Optimization of the E-Vehicle Route Profile*, SAE Technical Paper, No. 2016-01-0156.



## Appendix

The continuous state space model can be integrated into form

$$x(t) = e^{A(t-t_0)} x(t_0) + \int_0^T e^{A(t-\tau)} Bu(\tau) d\tau \quad (A1)$$

And using discrete time  $t = k.T$  the equation is reformulated into form

$$x(k+1) = e^{AT} x(k) + \int_0^T e^{A\tau} d\tau.Bu(\tau) d\tau \quad (A2)$$

Using first members of Taylor expansion for  $e^{A\tau} = I + A\tau = M$  and  $\int_0^T e^{A\tau} d\tau.B = IT.B = N$ , the discrete time model can be written into discrete state space form

$$\begin{aligned} x_{k+1} &= Mx_k + Nu_k \\ y_k &= Cx_k \end{aligned} \quad (A3)$$

The outputs in future time instants follows from recurrent substitution of previous equation on discrete time horizon  $N$ .

$$\begin{aligned} y_{k+1} &= Cx_{k+1} = C(Mx_k + Nu_k) \\ &\dots \\ y_{k+N} &= Cx_{k+N} = C(Mx_{k+N-1} + Nu_{k+N-1}) = \\ &= CM^N x_k + CM^{N-1}Nu_k + \dots + CNu_{k+N-1} \end{aligned} \quad (A4)$$

Introducing matrices

$$f = \begin{bmatrix} CM \\ CM^2 \\ \dots \\ CM^N \end{bmatrix} x_k = f_1 x_k \quad G = \begin{bmatrix} CN & \dots & 0 \\ \vdots & \ddots & \vdots \\ CM^{N-1}N & \dots & CN \end{bmatrix} \quad U = \begin{bmatrix} u_k \\ u_{k+1} \\ \dots \\ u_{k+N-1} \end{bmatrix} \quad (A5)$$

Outputs  $y_{k+i}$  and required outputs  $w_{k+i}$  can be written over whole prediction horizon  $N$  in the matrix form

$$Y = \begin{bmatrix} y_{k+1} \\ y_{k+2} \\ \dots \\ y_{k+N} \end{bmatrix} = f + GU, \quad w = \begin{bmatrix} w_{k+1} \\ w_{k+2} \\ \dots \\ w_{k+N} \end{bmatrix} \quad (A6)$$

The criteria  $J_k$  can be written in the matrix form

$$J_k = (Y - W)^T Q(Y - W) + U^T R U \quad (A7)$$

Then for minimum of  $J_k$  following holds as long as input  $u_k$  is unlimited:

$$\frac{\partial J_k}{\partial U^T} = 0 \quad (A8)$$

And substituting from (4) and (5) and several treatments leads into control matrix for whole prediction horizon

$$U = (G^T Q G + R)^{-1} G^T Q (W - f) \quad (A9)$$

To choose only first control action at time instant  $k$ , we can, using zero and unit square matrices with size of number of inputs, form selection matrix

$$\begin{aligned} e_1 &= [I_i \quad 0_i \quad \dots \quad 0_i] \\ u_k &= e_1 u \end{aligned} \quad (A10)$$

Considering (4), the control action can be written as

$$u_k = e_1 (G^T Q G + R)^{-1} G^T Q (-f_1 x_k) + e_1 (G^T Q G + R)^{-1} G^T Q W \quad (A11)$$

Introduction of closed loop gain matrix  $K = e_1 (G^T Q G + R)^{-1} G^T Q f_1$  and feed forward part  $K_w = e_1 (G^T Q G + R)^{-1} G^T Q W$  yields control law, consisting of state feedback and feedthrough components

$$u_k = -K x_k + K_w \quad (A12)$$

So stability of the controlled system can be easily checked by condition for closed loop pole position for discrete system

$$|eig(M - NK)| < 1 \quad (A13)$$

The equation (12) does not hold for MPC control with constraints, introduced in linear form

$$HU + LY < b \quad (A14)$$

Which are solved by quadratic programming together with

$$u_k^* = \arg \min_u J_k \quad (A15)$$

In this case, (12) is only necessary stability condition, but not sufficient. There is no straightforward stability condition for such case. Approach based on numerically evaluated Lyapunov function (based on optimality criteria  $J_k$  extended into infinite time using discrete Ricatti equation) and its numerical time derivative can be used. Suitably large subspace of the state space must be selected.

### Used symbols

<i>Symbol</i>	<i>Unit</i>	<i>Description</i>
$a$	$\text{m.s}^{-2}$	acceleration (– for deceleration)
$a_a$	$\text{m.s}^{-2}$	acceleration in driven (motor active) mode
$a_d$	$\text{m.s}^{-2}$	deceleration in coast-down mode (absolute value)
$s_d$	m	Length of acceleration phase
$s_a$	m	Length of deceleration phase
$v_1$	$\text{m.s}^{-1}$	Section entry velocity
$v_2$	$\text{m.s}^{-1}$	Constant velocity in the middle phase of the section
$v_3$	$\text{m.s}^{-1}$	Velocity in the beginning of braking phase
$v_4$	$\text{m.s}^{-1}$	Section output velocity
$s_{coa}$	m	Length of coasting down phase
$s_{con}$	m	Length of phase with constant velocity
$J_k$	–	Optimality criterion for model predictive controller design
$y_k$	–	Vector of system outputs at discrete time k
$w_k$	–	Vector of required system outputs at discrete time k
$Q_y$	–	Optimality output weighting matrix
$R_y$	–	Optimality input weighting matrix
$T$	s	Sampling period
$M, N$		Discrete linear dynamic system matrices
$A, B, C, D$		Linear dynamic system matrices produced by linearisation
$A_n, B_n, C_n$		Modified system matrices for predictive control design
SOC		State of charge
HVAC		Heating, ventilating, air-conditioning
MPC		Model predictive control
PACA		Predictive adaptive control algorithm
VCU		Voltage control unit